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Robust LQR for time-inhomogeneous Markov jump switched linear systems ⋆

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Abstract: Markov jump switched linear systems (MJSLSs) are switched linear systems, where a switching signal is governed by a Markov decision process. We consider a polytopic time-inhomogeneous setting, where the transition probabilities (between the operational modes of the system) associated to each discrete action are varying over time, with variations that are arbitrary within a polytopic set. We present and solve for this class of systems the finite horizon optimal control problem.

Keywords: Markov decision processes, uncertainty, robust control, linear optimal regulators.

1. INTRODUCTION

Wireless networked control systems (see, for instance, Schenato et al. (2007), Tabbara et al. (2007), Hespanha et al. (2007), Gupta et al. (2009), Heemels et al. (2010), Pajic et al. (2011), Alur et al. (2011) and references therein for a general overview) have a wide spectrum of applications, ranging from smart grid to remote surgery, passing through industrial automation, environment monitoring, intelligent transportation, and unmanned aerial vehicles, to name few. The wireless communication channels used to convey information between sensors, actuators, and computational units are frequently subject to time-varying fading and interference, which may lead to packet losses. The stochastic process that rules packet dropouts can be modeled by the transition probability (TP) matrix of the Markov chain (see e.g. the finite-state Markov modeling of Rayleigh, Rician and Nakagami fading channels in Sadeghi et al. (2008) and references therein). In most real cases, such probabilities cannot be computed exactly and are time-varying: we take into account this aspect by assuming that the Markov chain is time-inhomogeneous, i.e. a Markov chain having its TP matrix varying over time, with variations that are arbitrary within a polytopic set of stochastic matrices. Thus, the discrete-time (time-inhomogeneous) Markov jump linear systems (MJLSs, Costa et al. (2005), Zacchia Lun et al. (2016)) represent a promising mathematical model to jointly take into account the dynamics of a physical plant and nonidealities of wireless communication such as packet losses (see Schenato et al. (2007), Gonçalves et al. (2010), Smarra et al. (2015)). A MJLS model however does not take into account the fact, that fading channels can partially be compensated for by adjusting the transmission power levels (see Goldsmith (2005)); higher transmission power giving less packet errors, but increasing the energy consumption and interference with other systems. The problem of optimal power management in NCSs has been studied in Gatsis et al. (2014), where a restricted information structure on plant inputs and transmit powers was imposed, allowing them to be designed separately; it was shown that the optimal communication policy follows a Markov decision process (MDP) minimizing transmit power at the sensor and state estimation error at the controller. Considering a MDP instead of a Markov chain in MJLS brings to light a new type of system, that we call Markov jump switched linear system (MJSLS), which provides a mathematical framework for studying optimal power management for MJLS without any restrictions on continuous plant control inputs and discrete switching control policies regulating the transmission power values. The MJSLS model turns out to be rather general. It permits us to solve the finite-horizon robust (to uncertainties due to unknown, but bounded time-varying TPs) linear quadratic regulation (LQR) problem, where we jointly minimize costs of continuous and discrete control inputs, as a *main contribution* of this paper. It is worth noting that the proposed formulation can be applied to other co-design problems, such as a joint design of controller, routing and network coding in wireless control networks, presented in Di Girolamo et al. (2015). A vaguely similar model, which uses time-homogeneous Markov chains and discrete switching control without any associated discrete cost, has been considered in Vargas et al. (2010), where only a sub-optimal solution is provided based on a conservative approximation. To the best of our knowledge, we are the first to consider a polytopic time-inhomogeneous MJSLS model and to find the optimal analytical solution to the robust LQR problem.

The *notation* used throughout is standard. The sets of all positive and nonnegative integers are represented by $\mathbb{N}$ and $\mathbb{N}_0$, respectively. The set of all nonnegative reals is denoted by $\mathbb{R}_+$. The set of either reals $\mathbb{R}$ or complex numbers $\mathbb{C}$ is indicated by $\mathbb{F}$. The n-dimensional (either real or complex)

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Euclidean space is represented by $\mathbb{F}^n$, while a set of linear maps between two Euclidean spaces $\mathbb{F}^m$ and $\mathbb{F}^n$ is denoted by $\mathbb{F}^{m\times n}$ and is encoded through a set of $m\times n$ matrices. The conjugate of a complex matrix A is represented by $\bar{A}$, while the superscript * stands for the conjugate transpose of a matrix, and ^{T} indicates the transpose. Clearly, for a set of real matrices, * and ^{T} have the same meaning. We indicate with $\mathbb{C}^{n\times n}_*$ (respectively with $\mathbb{R}^{n\times n}_*$) the set of Hermitian (respectively symmetric) matrices, and with $\mathbb{F}^{n\times n}_+$ the set of positive semi-definite matrices. Unless otherwise stated, $\|\cdot\|$ will represent any norm in $\mathbb{F}^n$, and, for $A\in\mathbb{F}^{m\times n}$, $\|A\|$ will denote the induced uniform norm in $\mathbb{F}^{m\times n}$. The linear space made up of all N sequences of matrices $\mathbf{A}=[A_1,...,A_N]$, $A_i\in\mathbb{F}^{n\times m}$, $i\in\{1,\dots,N\}\subset\mathbb{N}$, is indicated by $\mathbb{H}^{m,n}$. For simplicity, we set $\mathbb{H}^n\triangleq\mathbb{H}^{n,n}$. For $\mathbf{A}\in\mathbb{H}^{m,n}$, we write $\mathbf{A}^*=[A_1^*,\dots,A_N^*]\in\mathbb{H}^{n,m}$, and say that $\mathbf{A}\in\mathbb{H}^n$ is Hermitian if $\mathbf{A}=\mathbf{A}^*$. We set $\mathbb{H}^{n*}\triangleq\{\mathbf{A}=[A_1,\dots,A_N]\in\mathbb{H}^n;\ A_i=A_i^*,\ i\in\{1,\dots,N\}\}$, $\mathbb{H}^{n+}\triangleq\{\mathbf{A}=[A_1,\dots,A_N]\in\mathbb{H}^{n*};\ A_i\succeq 0,\ i\in\{1,\dots,N\}\}$, where $A_i\succeq 0$ (respectively $A_i\succ 0$) indicates that A_i is positive semi-definite (respectively positive definite). For $\mathbf{A},\mathbf{B}\in\mathbb{H}^n$, if $\mathbf{A}-\mathbf{B}=[A_1-B_1,\dots,A_N-B_N]\in\mathbb{H}^{n+}$, we write $\mathbf{A}\succeq\mathbf{B}$. Similarly, the notation $\mathbf{A}\succ\mathbf{B}$ symbolizes that $A_i-B_i\succ 0$, i.e. the resulting matrices are positive definite $\forall i\in\{1,\dots,N\}\subset\mathbb{N}$. Finally, $\mathbb{I}_n$ denotes the $n\times n$ identity matrix, $\mathbf{E}[\cdot]$ stands for the mathematical expectation of the underlying scalar valued random variables, and $|\mathcal{S}|$ indicates cardinality of a finite set $\mathcal{S}$.

2. PROBLEM FORMULATION

Consider a probability space $(\Omega,\mathcal{F},\Pr)$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of (Borel) measurable events and Pr is the probability measure. Let a tuple $(\mathcal{S},\mathcal{A},p,g,s_0)$ be a Markov decision process defined on the probability space, where $\mathcal{S}=\{1,\dots,N\}$, $N\in\mathbb{N}$, is a finite set of discrete states; $\mathcal{A}=\{1,\dots,M\}$, $M\in\mathbb{N}$, is a finite set of actions (with slight abuse of notations we also define the function $\mathcal{A}(i)$, where $i\in\mathcal{S}$ and $\mathcal{A}(i)\subseteq\mathcal{A}$, to represent the available actions at state i); therefore, $p:\mathcal{S}\times\mathcal{A}\times\mathcal{S}\to[0,1]$ is the *transition probability* (TP) function, such that, for every decision epoch $k\in\mathcal{K}=\{0,\dots,T\}$, where $T\in\mathbb{N}_0$ is a finite control horizon, $i,j\in\mathcal{S}$, $a\in\mathcal{A}$,

$$p^a_{ij}(k)\triangleq\Pr\{s(k+1)=j\mid a,s(k)=i\}\geq 0, \tag{1a}$$

$$\sum\nolimits_{j=1}^{N}p^a_{ij}(k)=1 \text{ if } a\in\mathcal{A}(i),\quad p^a_{ij}(k)=0 \text{ if } a\notin\mathcal{A}(i); \tag{1b}$$

$g:\mathcal{S}\times\mathcal{A}\to\mathbb{R}_+$ is such that $g(i,a)$ is the (non-negative) cost when action $a\in\mathcal{A}(i)$ is taken; finally, $s_0\in\mathcal{S}$ is the initial state. We define the discrete state action function $\mu:\mathcal{S}\times\mathbb{N}_0\to\mathcal{A}$ such that $\mu(i,k)\in\mathcal{A}(i)$, $\forall i\in\mathcal{S}$, $k\in\mathcal{K}$. This function maps each state to the action selected (among those available at state i) at the decision epoch k. When i and k are obvious from the context, we will write μ instead of $\mu(i,k)$. The associated TP vector $\mathbf{p}^\mu_{i\bullet}(k)$ is a stochastic N-dimensional vector with entries $p^\mu_{ij}(k)$. In this work we assume that each $\mathbf{p}^\mu_{i\bullet}(k)$ is unknown and time-varying within a bounded set, $\forall\mu=\mu(i,k)\in\mathcal{A}(i)\subseteq\mathcal{A}$.

Assumption 1. The transition probability vector $\mathbf{p}^\mu_{i\bullet}(k)$ is **polytopic** $\forall\mu=\mu(i,k)\in\mathcal{A}(i)$, $i\in\mathcal{S}$, $k\in\mathcal{K}$, i.e.

$$\mathbf{p}^\mu_{i\bullet}(k)=\sum\nolimits_{l=1}^{L_\mu}\lambda_l(k)\mathbf{p}^\mu_{i\bullet|l},\ \lambda_l(k)\in\mathbb{R}_+,\ \sum\nolimits_{l=1}^{L_\mu}\lambda_l(k)=1, \tag{2}$$

$\forall k\in\mathbb{N}_0$, where $\{\mathbf{p}^\mu_{i\bullet|l}\}_{l\in\mathcal{L}_\mu\triangleq\{1,\dots,L_\mu\}}$, with $L_\mu\in\mathbb{N}$, is a given set of transition probability vectors, which are the vertices of a convex polytope for the action $\mu(i,k)\in\mathcal{A}(i)$.

Remark 2. The previous assumption is not restrictive, since the *polytopic uncertainty* described by its *vertices* is (widely) used for both robust control of MDPs with uncertain transition matrices (see e.g. Nilim and El Ghaoui (2005)) and for robust control of time-homogeneous MJLS (see e.g. Gonçalves et al. (2011), where the polytopic uncertainty model for TP matrices is additionally shown to be more general than the partly known element model). Moreover, also the interval TP matrix can be represented as a convex polytope, as described in Hartfiel (1998).

Let us consider the following discrete-time MJSLS $\mathbf{\Sigma}$,

$$\mathbf{\Sigma}\colon\ \begin{cases}x(k+1)=A_{s(k)}x(k)+B_{s(k)}u(k),\\ y(k)=C_{s(k)}x(k)+D_{s(k)}u(k),\\ x(0)=x_0,\ s(0)=s_0,\end{cases} \tag{3}$$

where $x(k)\in\mathbb{F}^n$ is a continuous state vector, $u(k)\in\mathbb{F}^m$ is a continuous control vector, $y(k)\in\mathbb{F}^s$ is a (measured) output of the system, $s(k)=i\in\mathcal{S}$ is a discrete state of MDP at time step $k\in\mathcal{K}$, and, $\forall i\in\mathcal{S}$, $A_i\in\mathbb{F}^{n\times n}$, $B_i\in\mathbb{F}^{n\times m}$, $C_i\in\mathbb{F}^{s\times n}$, $D_i\in\mathbb{F}^{s\times m}$ are constant state, input, output and direct transition (also known as feed-forward or feedthrough) matrices, respectively, each of which is associated with an operational mode of the system, while x_0 and s_0 are respectively continuous and discrete initial conditions. Considering every operational mode of a MJSLS $\mathbf{\Sigma}$, we have $\mathbf{A}=[A_1,\dots,A_N]\in\mathbb{H}^{n,n}$, $\mathbf{B}=[B_1,\dots,B_N]\in\mathbb{H}^{m,n}$, $\mathbf{C}=[C_1,\dots,C_N]\in\mathbb{H}^{n,s}$ and $\mathbf{D}=[D_1,\dots,D_N]\in\mathbb{H}^{m,s}$. In the following we denote by $c(k)$ the hybrid *control* pair $\{u(k),\mu(i,k)\}$, $k\in\mathbb{N}_0$. The sequence of pairs $\{c(k)\}_{k=0}^{T-1}$, i.e. $\{u(k),\mu(i,k)\}_{k=0}^{T-1}$ is called *hybrid control sequence*. At each time step (or decision epoch, in MDP terminology) k, a *particular choice* u_k of $u(k)$ is called the continuous control law; similarly, μ_k is denominated discrete switching control law. The pair $\{u_k,\mu_k\}$ forms the hybrid control *law*, and the sequence of hybrid control laws over the horizon T constitutes a finite horizon *feedback policy* $\boldsymbol{\pi}_T\triangleq\{u_k,\mu_k\}_{k=0}^{T-1}$. We cast a robust linear quadratic regulation (LQR) problem as a min-max problem of optimizing robust performance, i.e. finding the minimum over the finite horizon feedback policy of the maximum over the transition probability disturbance (obtained in correspondence of the chosen feedback policy).

Assumption 3. The continuous state variable $x(k)$ and the discrete operation mode $s(k)$ are known at each time k.

The random variables $\{x(t),s(t);t=0,\dots,k\}$ generate the σ-algebra denoted by $\mathcal{F}_k$. Clearly $\forall k\in\mathbb{N}_0$

$$\mathcal{F}_k\subset\mathcal{F}_{k+1}\subset\mathcal{F} \tag{4}$$

Remark 4. Requiring that all the continuous state variables are accessible to the controller may seem to be a strong assumption. However, the state-feedback scheme plays important role also in output-feedback setting, when the separation principle is applicable.

Assumption 5. Without loss of generality (Costa et al., 2005, p.74, Remark 4.1) we assume that for all $i\in\mathcal{S}$

$$C_i^*D_i=0, \tag{5}$$

$$D_i^*D_i\succ 0. \tag{6}$$

Problem 6. Given a MJSLS Σ having unknown and time-varying TP vectors as in Assumption 1, find for any

$k \in \{0, \ldots, T-1\}$ the optimal robust (with respect to uncertain transition probability) state-feedback policy $\boldsymbol{\pi}_T^\star = \{u_k^\star, \mu_k^\star\}_{k=0}^{T-1}$ that minimizes the following quadratic cost

$$\mathcal{J}(s_0, x_0, \boldsymbol{\pi}_T) \triangleq \max_{\mathbf{p}} \mathcal{J}(s_0, x_0, \boldsymbol{\pi}_T, \mathbf{p}), \quad (7)$$

$$\mathcal{J}(s_0, x_0, \boldsymbol{\pi}_T, \mathbf{p}) \triangleq \mathbf{E}\big[x^*(T) X_{s(T)}(T) x(T) + g_{s(T)} \mid \mathcal{F}_T\big] + \sum_{k=0}^{T-1} \big(\mathbf{E}\big[\|y(k)\|^2 + g(s(k), \mu(s(k), k)) \mid \mathcal{F}_0\big]\big), \quad (8)$$

where $\mathbf{p} \triangleq \{\mathbf{p}_{s(k)\bullet}^{\mu(s(k),k)}(k)\}_{k=0}^{T-1}$ is a transition probability sequence of length T obtained for the states $s(k) \in \mathcal{S}$ in correspondence of the switching actions $\mu(s(k), k)$; lastly, $\mathbf{X}(T) = (X_1(T), \ldots, X_N(T)) \in \mathbb{H}^{n+}$ is a vector of terminal continuous state weighs, which should be given for every discrete state in $\mathcal{S}$. Similarly, $\mathbf{g} = (g_1, \ldots, g_N) \in \mathbb{R}_+^N$ is a vector of terminal discrete state weights. The optimal quadratic cost will be indicated by $\mathcal{J}^\star(s_0, x_0)$.

3. ANALYTICAL SOLUTION

In this Section we address Problem 6 by providing an analytical solution based on a *dynamic programming*.
We follow the classical *Bellman optimization formulation* (see e.g. Bertsekas (1995)). We define a cost-to-go function at time k as $\mathcal{J}(s(k), x(k))$, such that

$$\mathcal{J}(s(k), x(k)) = \min_{c(k)} \mathcal{J}(s(k), x(k), c(k)), \quad (9)$$

$$\mathcal{J}(s(k), x(k), c(k)) = \max_{\mathbf{p}_{s(k)\bullet}^{\mu(s(k),k)}(k)} \mathcal{J}(s(k), x(k), c(k), \mathbf{p}_{s(k)\bullet}^{\mu(s(k),k)}(k)), \quad (10)$$

$$\mathcal{J}(s(k), x(k), c(k), \mathbf{p}_{s(k)\bullet}^{\mu(s(k),k)}(k)) = \quad (11)$$
$$\mathbf{E}[\|y(k)\|^2 + g(s(k), \mu(s(k), k)) + \mathcal{J}(s(k+1), x(k+1)) \mid \mathcal{F}_k],$$

$$\mathcal{J}(s(T), x(T)) = \mathbf{E}[x^*(T) X_{s(T)} x(T) + g_{s(T)} \mid \mathcal{F}_T]. \quad (12)$$

The explicit expressions of the cost-to-go function and of the optimal state-feedback policy are obtained by *backward induction*. Since $x(T)$ and $s(T)$ are $\mathcal{F}_T$-measurable, by linearity of the expected value, we have from (12) that

$$\mathcal{J}(s(T), x(T)) = x^*(T) X_{s(T)} x(T) + g_{s(T)}. \quad (13)$$

Thence, we examine the cost-to-go function at time step $k = T-1$. For every $i \in \mathcal{S}$, when $s(T-1) = i$, we apply (3), (5) and (13) to (11). By linearity of the expected value we get that the right-hand side (RHS) of (11) equals to

$$x^*(T-1) C_i^* C_i x(T-1) + u^*(T-1) D_i^* D_i u(T-1) +$$
$$x^*(T-1) A_i^* \mathbf{E}[X_{s(T)} \mid \mathcal{F}_{T-1}](A_i x(T-1) + B_i u(T-1)) +$$
$$u^*(T-1) B_i^* \mathbf{E}[X_{s(T)} \mid \mathcal{F}_{T-1}](A_i x(T-1) + B_i u(T-1)) +$$
$$g(i, \mu(i, T-1)) + \mathbf{E}[g_{s(T)} \mid \mathcal{F}_{T-1}], \quad (14)$$

since $C_i^* C_i$ and $D_i^* D_i$, A_i and B_i are constant matrices, $u(T-1)$ and $\mu(i, T-1)$ are (non-random) inputs we choose to apply to the system at time step $k = T-1$, and $x(T-1)$ is $\mathcal{F}_{T-1}$-measurable.
The explicit expression of $\mathbf{E}[X_{s(T)} \mid \mathcal{F}_{T-1}]$ is given by

$$\sum_{j=1}^N \mathbf{E}[\Pr\{s(T) = j \mid s(T-1) = i, \mu(i, T-1)\} X_j(T)] =$$
$$\sum_{j=1}^N p_{ij}^{\mu(i,T-1)}(T-1) X_j(T) = \mathbf{E}[X_{s(T)} \mid \mathcal{F}_{T-1}]. \quad (15)$$

Using the same procedure, we obtain

$$\mathbf{E}[g_{s(T)} \mid \mathcal{F}_{T-1}] = \sum_{j=1}^N p_{ij}^{\mu(i,T-1)}(T-1) g_j. \quad (16)$$

Since $\mathbf{X}(T) \in \mathbb{H}^{n+}$, it follows that $\mathbf{X}^*(T) = \mathbf{X}(T)$. Thus, from (14), (15) and (16), we obtain another expression for the RHS of (11), when $k = T-1$:

$$x^*(T-1)[C_i^* C_i + A_i^* \sum_{j=1}^N p_{ij}^{\mu(i,T-1)}(T-1) X_j(T) A_i] x(T-1) +$$
$$2x^*(T-1) A_i^* \sum_{j=1}^N p_{ij}^{\mu(i,T-1)}(T-1) X_j(T) B_i u(T-1) +$$
$$u^*(T-1)[D_i^* D_i + B_i^* \sum_{j=1}^N p_{ij}^{\mu(i,T-1)}(T-1) X_j(T) B_i] u(T-1) +$$
$$g(i, \mu(i, T-1)) + \sum_{j=1}^N p_{ij}^{\mu(i,T-1)}(T-1) g_j. \quad (17)$$

From (10) we see that its left-hand side (LHS) for $k = T-1$ is obtained from (17) by considering it as a function of only $\mathbf{p}_{i\bullet}^{\mu(i,T-1)}(T-1)$. For any choice of $\mu(i, T-1)$, from Assumption 1 we have that $\mathbf{p}_{i\bullet}^{\mu(i,T-1)}(T-1)$ is polytopic. It is immediate to verify that *Jensen's inequality* (Rockafellar, 1997, p. 25, Theorem 4.3) holds for (17) function of only $\mathbf{p}_{i\bullet}^{\mu(i,T-1)}(T-1)$, whatever it is $\mu(i, T-1)$, i.e. $\forall \lambda_l(T-1)$ defined in (2),

$$\mathcal{J}(i, x(T-1), c(T-1), \sum_{l=1}^{L_\mu} \lambda_l(T-1) \mathbf{p}_{i\bullet|l}^{\mu(i,T-1)}) =$$
$$\sum_{l=1}^{L_\mu} \lambda_l(T-1) \mathcal{J}(i, x(T-1), c(T-1), \mathbf{p}_{i\bullet|l}^{\mu(i,T-1)}).$$

Hence, $\mathcal{J}(i, x(T-1), c(T-1), \mathbf{p}_{i\bullet}^{\mu(i,T-1)}(T-1))$ is a *convex function* in variable $\mathbf{p}_{i\bullet}^{\mu(i,T-1)}(T-1)$, that belongs to a *polytopic set*. From (Rockafellar, 1997, p. 343, Theorem 32.2), this means that, for $l \in \mathcal{L}_{\mu(i,T-1)} = \{1, \ldots, L_{\mu(i,T-1)}\}$

$$\mathcal{J}(i, x(T-1), u(T-1), \mu(i, T-1)) = \quad (18)$$
$$\max_{\mathbf{p}_{i\bullet|l}^{\mu(i,T-1)}} \mathcal{J}(i, x(T-1), u(T-1), \mu(i, T-1), \mathbf{p}_{i\bullet|l}^{\mu(i,T-1)}).$$

In words, the *maximum in TPs of the cost-to-go function is attained on a vertex of the convex polytope of TPs.*
So, we examine the cost-to-go function on such vertices of convex polytope of TPs. We have, $\forall l \in \mathcal{L}_{\mu(i,T-1)}$, $k = T-1$, $s(k) = i$, $i \in \mathcal{S}$, that the LHS of (9) equals to

$$\min_{\substack{u(T-1), \\ \mu(i,T-1) \in \mathcal{A}}} \max_{\mathbf{p}_{i\bullet|l}^{\mu(i,T-1)}} \mathcal{J}\Big(i, x(T-1), u(T-1), \mu(i, T-1), \mathbf{p}_{i\bullet|l}^{\mu(i,T-1)}\Big), \quad (19)$$

with $\mathcal{J}(i, x(T-1), u(T-1), \mu(i, T-1), \mathbf{p}_{i\bullet|l}^{\mu(i,T-1)}) =$ (20)

$$x^*(T-1)[C_i^* C_i + A_i^* \sum_{j=1}^N p_{ij|l}^{\mu(i,T-1)} X_j(T) A_i] x(T-1) +$$
$$2x^*(T-1) A_i^* \sum_{j=1}^N p_{ij|l}^{\mu(i,T-1)} X_j(T) B_i u(T-1) +$$
$$u^*(T-1)[D_i^* D_i + B_i^* \sum_{j=1}^N p_{ij|l}^{\mu(i,T-1)} X_j(T) B_i] u(T-1) +$$
$$g(i, \mu(i, T-1)) + \sum_{j=1}^N p_{ij|l}^{\mu(i,T-1)} g_j.$$

We compute the minimum in $u(T-1)$ of (20) by equaling to 0 its derivative with respect to $u(T-1)$, obtaining that

$$2u^*(T-1)\ [D_i^* D_i + B_i^* \sum_{j=1}^N p_{ij|l}^{\mu(i,T-1)} X_j(T) B_i] +$$
$$2x^*(T-1)\ A_i^* \sum_{j=1}^N p_{ij|l}^{\mu(i,T-1)} X_j(T) B_i = 0.$$

Hence, it follows that the continuous control law is

$$u_{T-1} = K_{i|l}^{\mu_{T-1}} x(T-1),$$

$$K_{i|l}^{\mu_{T-1}} = -\left[D_i^* D_i + B_i^* \sum_{j=1}^{N} p_{ij|l}^{\mu_{T-1}} X_j(T) B_i\right]^{-1} \cdot B_i^* \sum_{j=1}^{N} p_{ij|l}^{\mu_{T-1}} X_j(T) A_i, \quad (21)$$

where μ_{T-1} is the switching control law of our choice. Considering that $\mathbf{p}_{i\bullet|l}^{\mu_{T-1}}$ depends on μ_{T-1}, we are dealing with a hierarchical decision problem, where the choice of the optimal switching control law $\mu_{T-1}^\star$ dominates the decision process. For any discrete state i, we can split our problem in a number (equal to $|\mathcal{A}(i)|$) of problems, where for each value of $\mu(i, T-1) = \mu$ we find

$$\min_{K_{i|l}^{\mu} x(T-1)} \max_{\mathbf{p}_{i\bullet|l}^{\mu}} \mathcal{J}(i, x(T-1), u(T-1) = K_{i|l}^{\mu} x(T-1), \mu, \mathbf{p}_{i\bullet|l}^{\mu}),$$

the smallest of which gives us the value of (19), and the related $\mu_{T-1}^\star$. Whatever it is $\mu_{T-1}^\star \in \mathcal{A}(i)$, when we choose it, the transition probability vector maximizing the cost-to-go function will be the corresponding vertex $\mathbf{p}_{i\bullet|l^\star}^{\mu_{T-1}^\star}$, where $l^\star$ is in $\mathcal{L}_{\mu_{T-1}^\star}$. The optimal continuous control law to apply will be $u_{T-1}^\star = K_{i|l^\star}^{\mu_{T-1}^\star} x(T-1)$. After applying the optimal hybrid control law in (20), we get that

$$\mathbf{p}_{i\bullet|l^\star}^{\mu_{T-1}^\star} = \arg \max_{\mathbf{p}_{i\bullet|l}^{\mu_{T-1}^\star}} \mathcal{J}(i, x(T-1), u_{T-1}^\star, \mu_{T-1}^\star, \mathbf{p}_{i\bullet|l}^{\mu_{T-1}^\star}),$$

$$\mathcal{J}(i, x(T-1), u_{T-1}^\star, \mu_{T-1}^\star, \mathbf{p}_{i\bullet|l^\star}^{\mu_{T-1}^\star}) \geq \mathcal{J}(i, x(T-1), u_{T-1}^\star, \mu_{T-1}^\star, \mathbf{p}_{i\bullet|l}^{\mu_{T-1}^\star}),$$

with equality valid for the transition probability of the vertex $l^\star$. This leads us to the following explicit expression

$$\begin{aligned}&\mathcal{J}(i, x(T-1)) = \mathcal{J}(i, x(T-1), u_{T-1}^\star, \mu_{T-1}^\star, \mathbf{p}_{i\bullet|l^\star}^{\mu_{T-1}^\star}) = \quad (22)\\ &x^*(T-1)[C_i^* C_i + A_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) A_i - \\ &A_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) B_i \left[D_i^* D_i + B_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) B_i\right]^{-1} \cdot \\ &B_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) A_i] x(T-1) + \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} g_j + g(i, \mu_{T-1}^\star) = \\ &x^*(T-1) X_i(T-1) x(T-1) + g_i(T-1), \quad (23)\end{aligned}$$

with $X_i(T-1) \triangleq X_i(T-1, \mu_{T-1}^\star, l^\star) \in \mathbb{F}_+^{n \times n}$, and with the cost of action $\mu_{T-1}^\star$ of $g_i(T-1) \triangleq g_i(T-1, \mu_{T-1}^\star, l^\star) \in \mathbb{R}_+$. In this way we obtain the first recursion of *coupled Riccati difference equations* (CRDEs), where $X_i(T-1)$ equals to

$$\begin{aligned}&X_i(T-1, \mu_{T-1}^\star, l^\star) = C_i^* C_i + A_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) A_i - \\ &A_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) B_i \left[D_i^* D_i + B_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) B_i\right]^{-1} \cdot \\ &B_i^* \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} X_j(T) A_i, \quad (24)\end{aligned}$$

(with $\mu_{T-1}^\star \in \mathcal{A}(i)$, $l^\star \in \mathcal{L}_{\mu_{T-1}^\star}$). We remark that apart from their dependency on $\mu_{T-1}^\star$ and $l^\star$, the CRDEs (24) have the same form of the ones presented in (Costa et al., 2005, p. 75). The first recursion for the cost $g_i(T-1)$ of the switching control law is

$$g_i(T-1, \mu_{T-1}^\star, l^\star) = \sum\nolimits_{j=1}^{N} p_{ij|l^\star}^{\mu_{T-1}^\star} g_j + g(i, \mu_{T-1}^\star) \quad (25)$$

From the explicit expressions of (22), provided in (23), (24), and (25), we see that the value of the cost-to-go function $\mathcal{J}(i, x(T-1))$ at time step $k = T-1$ depends on both discrete and continuous states of the system. Without knowing a priori the value of $x(T-1)$, we can not choose the optimal continuous state-feedback control gain $K_{i|l^\star}^{\mu_{T-1}^\star}$, which is described by (21). Thus, we need to consider, and store in memory, all vertices of convex polytopes of transition probability vectors, which possibly attain the maximum for those switching control laws, that ever achieve the minimum for some continuous state. This procedure requires to solve (24) for every $l \in \mathcal{L}_{\mu_{T-1}}$ of every $\mu_{T-1} \in \mathcal{A}(i)$, then perform some tests, described in the next section, to remove all the redundant solutions, and store all the remaining ones for the next time step, $k = T-2$. These subsets of so-called *parsimonious* solutions will be indicated by $\mathcal{L}_{\mu_{T-1}}^\star \subseteq \mathcal{L}_{\mu_{T-1}}$ and $\mathcal{A}^\star(i) \subseteq \mathcal{A}(i)$.

Now, given the (parsimonious) set of solutions of the first recursion of coupled Riccati difference equations (24), comprised of all $X_{s(T-1)}(T-1, \mu_{T-1}, l)$, such that $l \in \mathcal{L}_{\mu_{T-1}}^\star$ and $\mu_{T-1} \in \mathcal{A}^\star(i)$, let us examine (11) at time step $k = T-2$. From the same considerations used to obtain (14), $\forall i \in \mathcal{S}$, when $s(T-2) = i$, and for any given μ_{T-1}, l, such that $X_{s(T-1)}(T-1, \mu_{T-1}, l) = X_{s(T-1)}(T-1)$, $g_{s(T-1)}(T-1) = g_{s(T-1)}(T-1, \mu_{T-1}, l)$, we find that the RHS of (11), for $k = T-2$, equals to

$$\begin{aligned}&x^*(T-2) C_i^* C_i x(T-2) + u^*(T-2) D_i^* D_i u(T-2) + \\ &x^*(T-2) A_i^* \mathbf{E}[X_{s(T-1)}(T-1) | \mathcal{F}_{T-2}](A_i x(T-2) + B_i u(T-2)) + \\ &u^*(T-2) B_i^* \mathbf{E}[X_{s(T-1)}(T-1) | \mathcal{F}_{T-2}](A_i x(T-2) + B_i u(T-2)) + \\ &g(i, \mu(i, T-2)) + \mathbf{E}[g_{s(T-1)}(T-1) | \mathcal{F}_{T-2}], \quad (26)\end{aligned}$$

$$\mathbf{E}[X_{s(T-1)}(T-1) | \mathcal{F}_{T-2}] = \sum\nolimits_{j=1}^{N} p_{ij}^{\mu(i,T-2)}(T-2) X_j(T-1), \quad (27)$$

$$\mathbf{E}[g_{s(T-1)}(T-1) | \mathcal{F}_{T-2}] = \sum\nolimits_{j=1}^{N} p_{ij}^{\mu(i,T-2)}(T-2) g_j(T-1). \quad (28)$$

Notice, that the above expectations require considering all N possible discrete states of the system. Thus, we need to compute all possible permutations (with repetition) of parsimonious vertices $l \in \mathcal{L}_{\mu_{T-1}}^\star$ and actions $\mu_{T-1} \in \mathcal{A}^\star(j)$, for every $j \in \mathcal{S}$. Let us index (26) by $\xi(k) \in \Xi(k) \subset \mathbb{N}$. Then,

$$|\Xi(T-1)| = 1, \quad (29)$$

$$|\Xi(T-2)| = \prod\nolimits_{j=1}^{N} \prod\nolimits_{\iota=1}^{|\mathcal{A}^\star(j)|} |\mathcal{L}_{\mu_{T-1}(\iota)}^\star|, \quad (30)$$

$$|\Xi(T-2)|_{\max} = (ML)^N, \quad (31)$$

where $|\Xi(T-2)|_{\max}$ is the the maximum number of equations we need to solve at time step $k = T-2$, if all M actions are available for every discrete state, and there are no redundant solutions to CRDEs (24). In what follows, when the value of k will be clear from the context, we will write ξ instead of $\xi(k)$. For any $\xi(T-2) \in \Xi(T-2)$, we pursue the same line of reasoning applied for the previous time step. We have that $\mathcal{J}_\xi(i, x(T-2), u(T-2), \mu(i, T-2))$ is obtained from the related (26) by considering it as a function of only $\mathbf{p}_{i\bullet}^{\mu(i,T-2)}(T-2)$, which belongs to a polytopic set. As Jensen's inequality holds for (26), this function is convex in $\mathbf{p}_{i\bullet}^{\mu(i,T-2)}(T-2)$. So, from (Rockafellar, 1997, p. 343, Theorem 32.2), we obtain that

$$\begin{aligned}&\mathcal{J}_\xi(i, x(T-2), u(T-2), \mu(i, T-2)) = \quad (32)\\ &\max_{\mathbf{p}_{i\bullet|v}^{\mu(i,T-2)}} \mathcal{J}_\xi(i, x(T-2), u(T-2), \mu(i, T-2), \mathbf{p}_{i\bullet|v}^{\mu(i,T-2)}),\end{aligned}$$

with $v \in \mathcal{L}_{\mu(i,T-2)}$.

Applying the same steps as before, we find, that

$$u^{\xi}_{T-2} = K^{\mu_{T-2}}_{i|v,\xi} x(T-2), \tag{33}$$

$$K^{\mu_{T-2}}_{i|v,\xi} = -\left[D_i^* D_i + B_i^* \sum_{j=1}^{N} p^{\mu_{T-2}}_{ij|v} X_j(T-1) B_i\right]^{-1} \cdot B_i^* \sum_{j=1}^{N} p^{\mu_{T-2}}_{ij|v} X_j(T-1) A_i, \tag{34}$$

$$X_i^{\xi}(T-2, \mu^{\star}_{T-2}, v^{\star}) = C_i^* C_i + A_i^* \sum_{j=1}^{N} p^{\mu^{\star}_{T-2}}_{ij|v^{\star}} X_j(T-1) A_i + A_i^* \sum_{j=1}^{N} p^{\mu^{\star}_{T-2}}_{ij|v^{\star}} X_j(T-1) B_i K^{\mu_{T-2}}_{i|v,\xi}, \tag{35}$$

with $\xi = \xi(T-2) \in \Xi(T-2)$, $\mu^{\star}_{T-2} \in \mathcal{A}(i)$, $v^{\star} \in \mathcal{L}_{\mu^{\star}_{T-2}}$. Then,

$$g_i^{\xi}(T-2, \mu^{\star}_{T-2}, v^{\star}) = \sum_{j=1}^{N} p^{\mu^{\star}_{T-2}}_{ij|v^{\star}} g_j(T-1) + g(i, \mu^{\star}_{T-2}), \tag{36}$$

$$\mathcal{J}_{\xi}(i, x(T-2)) = x^*(T-2) X_i^{\xi}(T-2) x(T-2) + g_i^{\xi}(T-2). \tag{37}$$

As before, at the end of time step, for every $v \in \mathcal{L}_{\mu_{T-2}}$, we evaluate everyone of $|\Xi(T-2)|$ solutions of (35), and discard all the redundant solutions, in order to obtain smaller, parsimonious sets $\mathcal{L}^{\star}_{\mu_{T-2}} \subseteq \mathcal{L}_{\mu_{T-2}}$ and $\mathcal{A}^{\star}(i) \subseteq \mathcal{A}(i)$.
By iterating this procedure for a genetic $k = T-t$, with $t \in \{2, \dots, T\}$, it is possible to obtain the following:

$$|\Xi(k)| = \prod_{j=1}^{N} \prod_{\iota=1}^{|\mathcal{A}^{\star}(j)|} |\mathcal{L}^{\star}_{\mu_{k+1}(\iota)}|, \tag{38}$$

$$|\Xi(k)|_{\max} = M^{N^{t-1}} L^{\sum_{h=1}^{t-1} N^h}, \tag{39}$$

where $|\Xi(k)|_{\max}$ is the the maximum number of equations we need to solve, if all M actions are available for every discrete state, and there are no redundant solutions to CRDEs at any previous time step.

Theorem 7. Given a discrete-time Markov jump switched linear system Σ, described by (3), with Assumptions 1, 3, and 5 satisfied, for each initial condition x_0 and s_0, vectors of terminal continuous state weighs $\mathbf{X}(T)$, and discrete state weights $\mathbf{g}$, the optimal solution of Problem 6 is given by the policy $\boldsymbol{\pi}^{\star}_T$, having optimal switching control law

$$\mu^{\star}_k \in \mathcal{A}^{\star}(i), \tag{40}$$

such that $\forall k \in \mathbb{N}_0$, $k \leq T$, $\forall \xi \in \Xi(k)$ (with cardinality provided by (38)), $l \in \mathcal{L}^{\star}_{\mu_k}$, $\mu_k \in \mathcal{A}^{\star}(i)$, for any given $x(k)$,

$$\mathcal{J}(i, x(k)) = \min \mathcal{J}_{\xi}(i, x(k)), \tag{41}$$

$$\mathcal{J}_{\xi}(i, x(k)) = x^*(k) X_i^{\xi}(k, \mu_k, l) x(k) + g_i^{\xi}(k, \mu_k, l), \tag{42}$$

$$X_i^{\xi}(k, \mu_k, l) = C_i^* C_i + A_i^* \sum_{j=1}^{N} p^{\mu_k}_{ij|l} X_j(k+1) A_i + A_i^* \sum_{j=1}^{N} p^{\mu_k}_{ij|l} X_j(k+1) B_i K^{\mu_k}_{i|l,\xi}, \tag{43}$$

$$K^{\mu_k}_{i|l,\xi} = -\left[D_i^* D_i + B_i^* \sum_{j=1}^{N} p^{\mu_k}_{ij|l} X_j(k+1) B_i\right]^{-1} \cdot B_i^* \sum_{j=1}^{N} p^{\mu_k}_{ij|l} X_j(k+1) A_i, \tag{44}$$

$$g_i^{\xi}(k, \mu_k, l) = \sum_{j=1}^{N} p^{\mu_k}_{ij|l} g_j(k+1) + g(i, \mu_k), \tag{45}$$

and corresponding optimal continuous control law

$$u^{\star}_k = K^{\mu^{\star}_k}_{i|l,\xi} x(k). \tag{46}$$

where $\mathcal{A}^{\star}(i)$ and $\mathcal{L}^{\star}_{\mu_k}$ indicate the parsimonious sets, leading to the state-dependent optimal solutions.

Proof. By backward induction, it follows the procedure presented in the preceding part of this Section. □

4. ANALYTICAL REDUNDANCY

We have seen in the previous section that the value of the cost-to-go function (41) explicitly depends on the continuous state of the system in the CRDE part of the expression. However, not every cost-to-go function will be optimal, since, for instance, for any given $\mu_k \in \mathcal{A}(i)$, there is a possibility, that some costs will be smaller than others in TPs, for all continuous states $x(k)$. To define this concept formally, let us indicate the set of all possible cost-to-go candidates by $\Theta(k)$, $|\Theta(k)| \leq |\Xi(k)|_{\max}$. Also, let us denote by $\Xi(k) \subseteq \Theta(k)$ the set of optimal solutions. Clearly, if for any given $\mu_k \in \mathcal{A}(i)$, $\mathcal{J}_{\theta}(i, x(k)) \leq \mathcal{J}_{\xi}(i, x(k))$, $\forall x(k)$, where $\theta \in \Theta(k)$, $\xi \in \Xi(k)$, then $\mathcal{J}_{\theta}(i, x(k))$ cannot achieve the maximum in TPs associated with the switching law μ_k, and thus $\mathcal{J}_{\theta}(i, x(k))$ is redundant, and can be discarded. If we write both cost functions in their explicit form through (42), we get that $\mathcal{J}_{\theta}(i, x(k))$ is redundant, if for $l, v \in \mathcal{L}_{\mu_k}$

$$x^*(k)\Big(X_i^{\xi}(k, \mu_k, l) - X_i^{\theta}(k, \mu_k, v)\Big) x(k) \geq g_i^{\theta}(k, \mu_k, v) - g_i^{\xi}(k, \mu_k, l).$$

It is well a known fact, that if $X_i^{\xi}(k, \mu_k, l) \succeq X_i^{\theta}(k, \mu_k, v)$, then $x^*(k)(X_i^{\xi}(k, \mu_k, l) - X_i^{\theta}(k, \mu_k, v)) x(k) \geq 0$, $\forall x(k) \in \mathbb{F}^n$ (by definition of semi-definiteness). So, the first (sufficient) criterion, that permits us to find a redundant cost $\mathcal{J}_{\theta}(i, x(k))$ is the following:

$$\begin{aligned} &\text{if } g_i^{\xi}(k, \mu_k, l) - g_i^{\theta}(k, \mu_k, v) \geq 0 \;\&\; X_i^{\xi}(k, \mu_k, l) \succeq X_i^{\theta}(k, \mu_k, v) \\ &\text{then } \mathcal{J}_{\theta}(i, x(k)) \text{ is redundant: remove } \mathcal{J}_{\theta}, \theta(k), v \end{aligned} \tag{47}$$

The second criterion, always for a given $\mu_k \in \mathcal{A}(i)$, is computationally expensive, and is based on a fact, that if the $\mathcal{J}_{\theta}(i, x(k))$ is a convex combination of other cost-to-go candidates $\mathcal{J}_{\xi_{\iota}}(i, x(k))$, where $\xi_{\iota} \in \Xi(k)$, $l_{\iota} \in \mathcal{L}_{\mu_k} \backslash v$, then $\mathcal{J}_{\theta}(i, x(k))$ is an interior point of a convex polytope defined by those $\mathcal{J}_{\xi_{\iota}}(i, x(k))$ as vertices; by (Rockafellar, 1997, p. 343, Theorem 32.2), $\mathcal{J}_{\theta}(i, x(k))$ cannot attain the maximum in transition probabilities. Formally, we write that, for any given discrete state $i \in \mathcal{S}$ and any discrete switching law $\mu_k \in \mathcal{A}(i)$, if there $\exists \alpha_{\iota} \in \mathbb{R}_+$, $\iota \in \mathbb{N}$, such that $\sum_{\iota} \alpha_{\iota} = 1$, $\mathcal{J}_{\theta}(i, x(k)) = \sum_{\iota} \alpha_{\iota} \mathcal{J}_{\xi_{\iota}}(i, x(k))$, $\forall x(k)$, and $\iota \leq L_{\mu_k} - 1$, then $\mathcal{J}_{\theta}(i, x(k))$ is redundant. In practice, this implies verifying, whether

$$g_i^{\theta}(k, \mu_k, v) = \sum_{\iota} \alpha_{\iota} g_i^{\xi_{\iota}}(k, \mu_k, l) \;\&\; X_i^{\theta}(k, \mu_k, v) = \sum_{\iota} \alpha_{\iota} X_i^{\xi_{\iota}}(k, \mu_k, l_{\iota}), \tag{48}$$

which requires solving a system of linear equations and checking if the solution gives us a convex combination. As a side note, we can combine (47) and (48) in a unique, more general, but also more complex to check, criterion. After removing all the redundant solutions from $\Theta(k)$ via (47) and (48), we obtain a smaller set, sometimes called *parsimonious* (see Lincoln and Rantzer (2006)), of the cost-to-go functions (42), which can ever achieve the maximum in transition probabilities for some continuous state $x(k)$. For every $\mu_k \in \mathcal{A}(i)$, we denote the corresponding parsimonious set of indices of vertices of a convex polytope of TPs by $\mathcal{L}^{\star}_{\mu_k}$. As a final note, we observe that, if for every $x(k)$, every cost-to-go function (42) correspondent to vertices of a convex polytope of TPs for a switching control law μ_k, is smaller than every cost-to-go candidate function (42) correspondent to vertices of a convex polytope of TPs for another switching control law μ'_k, then μ'_k is redundant at time step k. Formally, we write that

$$\text{if } \forall l \in \mathcal{L}^\star_{\mu_k},\ v \in \mathcal{L}^\star_{\mu'_k},\ g^\theta_i(k,\mu'_k,v) - g^\xi_i(k,\mu_k,l) \geq 0\ \&$$
$$X^\theta_i(k,\mu'_k,v) \succeq X^\xi_i(k,\mu_k,l),$$
$$\text{then } \mu'_k \text{ is redundant: remove all related } \mathcal{J}_\theta,\ \theta(k),\ v. \quad (49)$$

The set of parsimonious switching laws, obtained after discarding all redundant ones, is indicated by $\mathcal{A}^\star(i)$.

5. CONCLUSIONS

The discrete-time time-inhomogeneous Markov(ian) jump switched linear systems (abbreviated to MJSLSs) with polytopic uncertainties on transition probabilities (TPs) introduced in this work represent a promising mathematical model of linear systems subject to abrupt parameter changes (due, for instance, to abrupt environmental disturbances, or to changes in subsystems interconnections), which can partially be compensated for via an additional discrete switching control action. The polytopic time-inhomogeneous (PTI) setting permits us to take into account the fact that in most real cases the TPs between operational modes of the system cannot be computed exactly and are time-varying. An example of such scenario can be found in optimal power management problem for wireless networked control systems represented via Markov jump linear system (MJLS) model, where the choice of the transmission power influences the pocket error probabilities, together with energy consumption and interference with other systems.

In the presented work we solved the optimal state-feedback control problem for MJSLSs with PTI transition probabilities between operational modes. The proposed solution is robust to such time-varying uncertainties in TPs. It also solves the problems of the optimal state-feedback control in time-homogeneous setting (when there is only one TP matrix, which is known at each time step) and for MJLSs (where there is no switching control actions available).

We plan to extend our work to the quadratic optimal control with partial information on continuous and discrete system states, taking into account also the process and measurement noises. The infinite horizon problem and the time-delayed observations of the discrete system states are other open topics we intend to look into.

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