Robust Linear Quadratic Regulation Over Polytopic Time-Inhomogeneous Markovian Channels Under Generalized Packet Dropout Compensation

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***Abstract*—This letter addresses a fundamental issue of time-varying parametric uncertainty affecting unreliable communication links that convey the control commands to actuators in wireless networked control systems. It introduces the polytopic time-inhomogeneous finite-state Markov channel model to account for significant changes in possible propagation channel characteristics and analytically solves the linear quadratic regulation problem under the generalized packet dropout compensation. An example validating the results is presented.**

***Index Terms*—Control over communications, Markov processes, robust control.**

I. Introduction

WIRELESS networked control systems support countless mission-critical applications in industrial automation, intelligent transportation, remote surgery, and smart grids. See, e.g., [1], [2], [3], and [4] as an overview of significant recent advances in the related research. A fundamental topic in wireless networked control research is estimation and control over unreliable links, explored, e.g., in [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], and [16].

The finite-state Markov channel (FSMC, see, e.g., [17] and [18]) is a simple yet powerful analytical model capturing the nominal dynamics of wireless links. These links are strongly affected by environmental factors such as physical obstacles and electromagnetic interference that can change significantly when transmitters, receivers, or both are mobile. To account for the resulting time-varying parametric uncertainty due to environmental factors and random and systematic errors of measurement and numerical computation procedures, we introduce a polytopic time-inhomogeneous (PTI) characterization of FSMCs inspired by the PTI models used in the robust control of Markov jump systems, as presented, e.g., in [19], [20], and [21]. The PTI FSMC yields significant complexity explicitly addressed in this letter.

An additional source of complexity is the generalized control-packet dropout compensation (GC-PDC), requiring actuators to apply an appropriately scaled last available control input when the communication link (CL) corrupts the received control command. This approach includes both zero-input and hold-input PDC as notable cases and is particularly suitable for less performant actuators that cannot zero the control inputs immediately. This letter fundamentally differs from previous works on GC-PDC, such as [13], [22], and [23], in considering the PTI FSMC to govern the packet dropout dynamics instead of a simple Bernoulli process, resulting in a much more general and complex problem.

The *contribution* of this letter is twofold:

- It introduces the PTI FSMC model for robust control over unreliable CLs and proves through Lemmas 1–3 that the finite-state PTI structure is preserved under GC-PDC.
- It analytically solves in Theorem 1 the finite-horizon linear quadratic regulation (LQR) problem under PTI FSMC dynamics of CLs, one-time-step delayed channel state information, and generalized PDC and explicitly addresses its intrinsic complexity.

Under GC-PDC combined with PTI FSMC dynamics of CLs, intermittent control inputs remain active for random periods, whose probability comes from the product of matrices within a polytopic set. Such a setting presents significant technical challenges that require a tailored model of the closed-loop system, careful handling of large growing set of coupled matrix equations, and a detailed analysis of the PTI FSMC properties involving even the joint spectral radius of matrix set to ensure the existence of a tractable finite-state model.

This letter is *organized* as follows. Section II presents the closed-loop system model accounting for white Gaussian process noise and generalized PDC, introduces the PTI FSMC representation of the CL dynamics, and formalizes the robust LQR problem in the finite horizon. Section III derives the structural properties of the considered system and presents the main results. Section IV validates them with a numerical example of an inverted pendulum on a cart. The final Section V concludes this letter with an outlook on future work.

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Notation: $\mathbb{R}$, $\mathbb{Z}^{+}$, and $\mathbb{Z}^{0+}$ denote the sets of reals and positive and nonnegative integers, respectively. $\mathbb{R}^n$ indicates the n-dimensional Euclidean space. The symmetric positive definite and positive semi-definite matrices M are denoted by $M \succ 0$ and $M \succeq 0$. Then, $M^\top$ indicates the transpose of a matrix M. Finally, $\mathbb{P}$ represents the probability of an event, and $\mathbb{E}$ denotes the expected value of a random variable.

II. System Model and Problem Definition

Consider a discrete-time linear system with intermittent control inputs from the remote controller due to lossy communication and generalized dropout compensation:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k, \\ u_k = \delta_k u_k^c + (1-\delta_k)\phi u_{k-1}, \end{cases} \tag{1}$$

where for all $k \in \mathbb{Z}^{0+}$, $x_k \in \mathbb{R}^{n_x}$ and $u_k \in \mathbb{R}^{n_u}$ are the real-valued system state and control input to actuators, A and B are state and input matrices of appropriate size, and $w_k \in \mathbb{R}^{n_x}$ is a white Gaussian process noise having zero mean and covariance matrix Σ_w. $\{\delta_k\}$ is a binary stochastic process that models the packet loss between the controller and the actuators, and $u_k^c \in \mathbb{R}^{n_u}$ is the updated desired control input sent by the remote controller. Upon correct delivery, $u_k = u_k^c$; otherwise, if the desired control input is corrupted or lost, the actuators apply a scaled version of an outdated control input, $u_k = \phi u_{k-1}$, with $\phi \in [0, 1]$. This generalized approach to control-packet dropout compensation covers both the zero-input ($\phi = 0$) and hold-input ($\phi = 1$) PDCs as notable cases.

Polytopic Markovian Channel Model: We model the dynamics of the control-packet loss process $\{\delta_k\}$ in the wireless propagation environment with the mobility of the transmitter, receiver, or both as a PTI FSMC.

This channel has a fixed finite set $\mathcal{S}$ of states $\{s_i\}_{i=1}^N$, each characterized by predefined signal-to-interference-plus-noise ratio (SINR) thresholds, which do not vary in time, so the state number remains constant, and each channel state has a specific packet dropout probability. A discrete-time Markov chain $\{\theta_k\}$ determines the active channel state, whose packet dropout probability and transition probabilities (TPs) depend on the SINR dynamics that may be time-varying due to the mobility of the transmitter, receiver, or both. Mobility may significantly change the propagation channel characteristics by varying, for instance, distances, movement velocities, and relative positioning of obstacles in the environment. When the variables defining the propagation channel characteristics cannot be timely and accurately measured, the exact values of the channel state transition and packet dropout probabilities are unknown. However, the general knowledge of the propagation environment and plant characteristics allows the system designer to identify the range of possible values of each relevant variable and use the PTI model. So, we formally define the PTI FSMC as follows. For all $k \in \mathbb{Z}^{0+}$,

$$\mathbb{P}\big(\theta_k = s_j \mid \theta_{k-1} = s_i\big) = p_{ij}(k) \geq 0, \ \sum_{j=1}^N p_{ij}(k) = 1, \tag{2}$$

$$\mathbb{P}\big(\delta_k = 1 \mid \theta_k = s_j\big) = \hat{\delta}_j(k), \ \mathbb{P}\big(\delta_k = 0 \mid \theta_k = s_j\big) = 1 - \hat{\delta}_j(k). \tag{3}$$

For notational convenience, group channel state TPs in the transition probability matrix (TPM) $P_c(k) \triangleq [p_{ij}(k)]_{i,j=1}^N$ and the success (or, conversely, failure) probabilities of delivering a packet at time k in the following matrices:

$$P_s(k) \triangleq \Big[p_{ij}(k)\hat{\delta}_j(k)\Big]_{i,j=1}^N, \ P_f(k) = P_c(k) - P_s(k). \tag{4}$$

Assumption 1: The matrices $P_s(k)$ and $P_f(k)$ are PTI, i.e., $\forall k \in \mathbb{Z}^{0+}$, $\exists \{\lambda_{v_l}(k)\}_{v_l=1}^{V_l}$ with $\lambda_{v_l}(k) \in \mathbb{R}$ such that

$$\lambda_{v_l}(k) \geq 0, \ \sum_{v_l=1}^{V_l} \lambda_{v_l}(k) = 1, \tag{5a}$$

where $l \in \{s, f\}$ and $V_l \in \mathbb{Z}^{+}$ so that

$$P_s(k) = \sum_{v_s=1}^{V_s} \lambda_{v_s}(k) P_{s.v_s}, \ P_f(k) = \sum_{v_f=1}^{V_f} \lambda_{v_f}(k) P_{f.v_f}, \tag{5b}$$

where $\{P_{s.v_s}\}_{v_s=1}^{V_s}$ and $\{P_{f.v_f}\}_{v_f=1}^{V_f}$ are given finite sets of matrices, which are the vertices of a convex polytopes, and $\lambda_{v_l}(k)$ are unmeasurable.

Problem Statement: We cast a robust LQR problem as a min-max problem of optimizing robust performance, i.e., finding the minimum over the finite-horizon state-feedback law of the maximum over the TP disturbance (obtained under the chosen feedback law). Similarly to [16], we make the following common technical assumptions (with $k \in \mathbb{Z}^{0+}$).

Assumption 2: The initial conditions (x_0, θ_0) are independent random variables. For any k, the white Gaussian process noise sequence $\{w_k\}$ is independent of the initial conditions x_0 and θ_0 and of the binary stochastic variable δ_k. The Markov chain $\{\theta_k\}$ and the noise sequence $\{w_k\}$ are independent, too.

As detailed in [16], [18], and references therein, the wireless receivers determine the message transmission outcome and FSMC state and feed this information back to the respective transmitters. In a wireless control system described by (1), the controller is the transmitter that thus can access the channel state information with one time-step delay. Hence, the information set of a state-feedback controller is

$$\mathcal{I}_k = \{(x_t)_{t=0}^k, (\delta_{t-1})_{t=1}^k, (\theta_{t-1})_{t=1}^k\}. \tag{6}$$

This letter solves the following optimization problem.

Problem 1: Given a system (1), find the optimal state-feedback control law $\check{\mathbf{u}}_T^c$ that minimizes the quadratic functional cost of the closed-loop system over a finite-time horizon T for the worst possible sequence of control message transmission success and failure probabilities under Assumptions 1 and 2 and information set (6).

Formally, let $\mathbf{u}_T^c \triangleq (u_k^c)_{k=0}^{T-1}$ be a control law defined by a sequence of FSMC-dependent state-feedback control inputs,

$$u_k^c = K_{(k,\theta_{k-1})} x_k, \tag{7}$$

and $\mathcal{U}_T$ be the set of all such control laws. Furthermore, for $l \in \{s, f\}$, let $P_{l|i\bullet}(k)$ indicate the ith row of $P_l(k)$ so that $\mathbf{P}_T \triangleq (P_{l|i\bullet}(t))_{t=1}^T$ denotes a control message transmission success and failure probability sequence of length T, and $\mathcal{P}_T$ indicates the set of all such sequences. Then,

$$\check{\mathbf{u}}_T^c = \arg\min_{\mathbf{u}_T^c \in \mathcal{U}_T} \max_{\mathbf{P}_T \in \mathcal{P}_T} J\big(x_0, \mathcal{U}_T, \mathcal{P}_T\big), \text{ where} \tag{8}$$

$$J\big(x_0, \mathcal{U}_T, \mathcal{P}_T\big) = \mathbb{E}\left(\sum_{k=0}^{T-1}(x_k^\top Q x_k + u_k^\top R u_k) + x_T^\top Q x_T \mid \mathcal{I}_0\right), \tag{9}$$

which is the cost function, and $Q \succeq 0$ and $R \succ 0$ are given state-weighting and input-weighting matrices, respectively. The cost (9) weights the inputs to actuators, thus accounting for the dropout compensation factor ϕ and the desired control law $\mathbf{u}_T^c \in \mathcal{U}_T$ and depending on the realizations of the stochastic process $\{\delta_k\}$, which obeys the PTI sequences $\mathbf{P}_T \in \mathcal{P}_T$.

III. Robust Linear Quadratic Regulation

To solve Problem 1 analytically, we first describe the system (1) in the time instances in which actuators successfully receive messages from controllers, then present the PTI structure of the related discrete state TPs, and finally exploit their structural properties.

System States in Packet Delivery Time Instances: Count the consecutive control message dropouts observed by a controller at time step k in a stochastic variable Δ_k.

$$\Delta_k = (1-\delta_{k-1})(\Delta_{k-1}+1). \tag{10}$$
$$\Delta_k = \ell \Leftrightarrow \delta_{k-1-\ell} = 1 \wedge \delta_{k-t} = 0 \ \forall t \in \mathbb{Z}^+ : t \le \ell. \tag{11}$$

Notably, if $\ell = 0$, then $\{t\}_{t=1}^0 = \emptyset$, meaning that (11) becomes $\Delta_k = 0 \Leftrightarrow \delta_{k-1} = 1$. Denote by $\mathcal{T}$ a set of time instances in which actuators successfully receive the controller's messages:

$$\mathcal{T} \triangleq \{k : \delta_k = 1\}_{k\in\mathbb{Z}^{0+}} = \{\tau_{(m)}\}_{m\in\mathbb{Z}^{0+}}. \tag{12a}$$

From (10), (11), and (12a), for all $m \in \mathbb{Z}^{0+}$,

$$\tau_{(m)} \in \mathcal{T} \Rightarrow \delta_{\tau_{(m)}} = 1 \Rightarrow \Delta_{\tau_{(m)}+1} = 0, \tag{12b}$$
$$\tau_{(m+1)} = \tau_{(m)} + 1 + \Delta_{\tau_{(m+1)}}. \tag{12c}$$

For the notational convenience, for any $k, n \in \mathbb{Z}^{0+}$, let

$$\Phi_{(n)} \triangleq \sum_{j=0}^{n} A^j B \phi^{n-j}, \quad W_{(k,n)} \triangleq \sum_{j=0}^{n} A^{n-j} w_{k+j}. \tag{13}$$

Then, from (10)–(13), the following equation describes the dynamics of the system (1) with control inputs as in (7):

$$\begin{cases} \Delta_{\tau_{(m+1)}} = n \in \mathbb{Z}^{0+} \Rightarrow \forall h \in \mathbb{Z}^{0+} : h \le n, \\ x_{\tau_{(m)}+1+h} = A^{h+1} x_{\tau_{(m)}} + \Phi_{(h)} u_{\tau_{(m)}} + W_{(\tau_{(m)},h)}, \\ u_{\tau_{(m)}+h} = \phi^h K_{\left(\tau_{(m)},\theta_{\tau_{(m)}-1}\right)} x_{\tau_{(m)}}. \end{cases} \tag{14}$$

System Model for LQR: To provide the stochastic characterization of the above system dynamics, we group the current duration of a packet error burst and the last known wireless channel state in one augmented discrete state: $\eta_k \triangleq (\Delta_k, \theta_{k-1})$. Then, from (12),

$$\eta_{\tau_{(m)}} = \left(\Delta_{\tau_{(m)}}, \theta_{\tau_{(m)}-1}\right), \tag{15a}$$
$$\eta_{\tau_{(m+1)}} = \left(\Delta_{\tau_{(m+1)}}, \theta_{\tau_{(m)}+\Delta_{\tau_{(m+1)}}}\right), \tag{15b}$$

where $\Delta_{\tau_{(m+1)}}$ indicates the time interval the transmitted control input may remain active. From the controller's perspective, $\eta_{\tau_{(m)}}$ is known, while $\eta_{\tau_{(m+1)}}$ is a random variable.

Lemma 1: For any pair of discrete states (15), it holds that

$$\mathbb{P}\left(\eta_{\tau_{(m+1)}} = (n, s_j) \mid \eta_{\tau_{(m)}} = (\ell, s_i)\right) = \frac{e_i^\top P_s\left(\tau_{(m)}\right) \prod_{t=1}^n P_f\left(\tau_{(m)}+t\right) e_j e_j^\top P_s\left(\tau_{(m+1)}\right)\mathbf{1}}{e_i^\top P_s\left(\tau_{(m)}\right)\mathbf{1}}, \tag{16}$$

where e_i and e_j denote the ith and jth column vectors of the standard basis of $\mathbb{R}^N$ (all their components are zero except the ith for e_i and jth for e_j, and the nonzero element equals one), and $\mathbf{1}$ indicates the column vector of length N with all components equal one.

Proof: It follows from (2)–(4), (10)–(12), the conditional probability definition, the chain rule of probability, the independence of both δ_k and θ_k of δ_{k-t} $\forall t \in \mathbb{Z}^+$, the Markov property, and the law of total probability. ■

Remark 1: TPs in (16) are independent of ℓ, i.e., the value of $\Delta_{\tau_{(m)}}$. So, (16) equals $\mathbb{P}(\eta_{\tau_{(m+1)}} = (n, s_j) \mid \theta_{\tau_{(m)}-1} = s_i)$.

For the notational convenience, let

$$\varsigma_{(\tau_{(m)},n,j)} \triangleq \prod_{t=1}^{n} P_f\left(\tau_{(m)}+t\right) e_j e_j^\top P_s\left(\tau_{(m+1)}\right)\mathbf{1} \tag{17a}$$

denote the vector of probabilities of an n-length packet error burst that ends with the FSMC being in state s_j, and

$$\zeta_{(\tau_{(m)},i,n,j)} \triangleq \frac{e_i^\top P_s\left(\tau_{(m)}\right)\varsigma_{(\tau_{(m)},n,j)}}{e_i^\top P_s\left(\tau_{(m)}\right)\mathbf{1}} \tag{17b}$$

be the TP as in (16).

Lemma 2: For all $\tau_{(m)} \in \mathbb{Z}^{0+}$, $i, j, N \in \mathbb{Z}^+$, $i, j \le N$, and an arbitrarily small threshold ϵ, under Assumption 1, there always exists a maximal number of consecutive control message dropouts L such that

$$L \triangleq \underset{\hat{n}\in\mathbb{Z}^+}{\arg\min}\, \zeta_{(\tau_{(m)},i,\hat{n}+1,j)} < \epsilon. \tag{18}$$

Proof: See Appendix. ■

We group the augmented discrete state unique TPs in the following compact TPM:

$$Z\left(\tau_{(m)}\right) \triangleq \left[\zeta_{(\tau_{(m)},i,n,j)}\right]_{i,\gamma(n,j)=1}^{N,(L+1)N}, \quad \gamma(n,j) \triangleq nN + j, \tag{19}$$

and constructing a complete TPM of (16) is unnecessary since it would just repeat each row of $Z(\tau_{(m)})$ L times.

Lemma 3: Under Assumption 1, $Z(k)$ is PTI $\forall k \in \mathbb{Z}^{0+}$.

Proof: See Appendix. ■

Remark 2: The number of vertices that define $Z(k)$ may be significantly smaller than $V_s^2 V_f^L$ since the matrix product of two vertices defining polytopic matrix sets may produce an interior point of a new set [24, Sec. 2.3]. We can obtain all the vertices of $Z(k)$ by recursively finding and removing from the vertex representation above the interior points (via, e.g., linear programming). Let $\{Z_{v_\zeta}\}_{v_\zeta=1}^{V_\zeta}$ denote the minimal set defining the vertex representation of $Z(k)$. Then,

$$Z(k) = \sum_{v_\zeta=1}^{V_\zeta} \hat{\lambda}_{v_\zeta}(k) Z_{v_\zeta}, \ Z_{v_\zeta} = \left[\zeta_{(i,n,j).v_\zeta}\right]_{i,\gamma(n,j)=1}^{N,(L+1)N}, \tag{20a}$$

$$\hat{\lambda}_{v_\zeta}(k) \ge 0, \ \sum_{v_\zeta=1}^{V_\zeta} \hat{\lambda}_{v_\zeta}(k) = 1, \ \zeta_{(i,n,j).v_\zeta} = \frac{e_i^\top P_{s.v_1} \varsigma_{(n,j).v_\zeta}}{e_i^\top P_{s.v_1}\mathbf{1}}, \tag{20b}$$

$$\varsigma_{(n,j).v_\zeta} = \prod_{t=1}^{n} P_{f.v_{t+1}} e_j e_j^\top P_{s.v_{n+2}}\mathbf{1} \tag{20c}$$

for all $k \in \mathbb{Z}^{0+}$ and some v_ι, with $\iota \in \mathbb{Z}^+$, $\iota \le n+2$, $n \in \mathbb{Z}^{0+}$, and $n \le L$, such that $P_{l.v_\iota} \in \{P_{l.v_l}\}_{v_l=1}^{V_l}$, $l \in \{s, f\}$.

Remark 3: The recursive derivation of the set $\{Z_{v_\zeta}\}_{v_\zeta=1}^{V_\zeta}$ does not require the prior knowledge of L: the recursion in n

stops once each component $\zeta_{(i,n+1,j).v_\zeta} < \epsilon\ \forall i,j \le N, v_\zeta \le V_\zeta$, and arbitrarily small ϵ, so $n = L$.

Analytical Solution of Problem 1

Theorem 1: For all $k \in \mathbb{Z}^{0+}$ such that $k < T$ and any value $s_i \in \mathcal{S}$ of θ_{k-1}, the solution to Problem 1 is the following.

$$K_{(k,s_i)} = -\mathcal{B}^{-1}_{(k,s_i).\varrho_k}\mathcal{C}_{(k,s_i).\varrho_k}, \tag{21a}$$

$$\mathcal{C}_{(k,s_i).\varrho_k} = \sum_{h=0}^{L-\xi_k} q_{(i,h).\hat{v}_\zeta}\sum_{r=1}^{h}\Phi^\top_{(r-1)}QA^r + \sum_{h=0}^{L-\xi_k}\sum_{j=1}^{N}\zeta_{(i,h,j).\hat{v}_\zeta}\Phi^\top_{(h)}\mathcal{X}_{(k+1+h,s_j).\varrho_{k+1+h}}A^{h+1}, \tag{21b}$$

$$\mathcal{B}_{(k,s_i).\varrho_k} = R + \sum_{h=0}^{L-\xi_k} q_{(i,h).\hat{v}_\zeta}\sum_{r=1}^{h}\phi^{2r}R + \sum_{h=0}^{L-\xi_k} q_{(i,h).\hat{v}_\zeta}\sum_{r=1}^{h}\Phi^\top_{(r-1)}Q\Phi_{(r-1)} + \sum_{h=0}^{L-\xi_k}\sum_{j=1}^{N}\zeta_{(i,h,j).\hat{v}_\zeta}\Phi^\top_{(h)}\mathcal{X}_{(k+1+h,s_j).\varrho_{k+1+h}}\Phi_{(h)}, \tag{21c}$$

$$\mathcal{X}_{(k,s_i).\varrho_k} = \mathcal{A}_{(k,s_i).\varrho_k} - \mathcal{C}^\top_{(k,s_i).\varrho_k}\mathcal{B}^{-1}_{(k,s_i).\varrho_k}\mathcal{C}_{(k,s_i).\varrho_k}, \tag{21d}$$

$$\mathcal{A}_{(k,s_i).\varrho_k} = Q + \sum_{h=0}^{L-\xi_k}\sum_{j=1}^{N}\zeta_{(i,h,j).\hat{v}_\zeta}\left(A^{h+1}\right)^\top\mathcal{X}_{(k+1+h,s_j).\varrho_{k+1+h}}A^{h+1} + \sum_{h=0}^{L-\xi_k} q_{(i,h).\hat{v}_\zeta}\sum_{r=1}^{h}A^{r\top}QA^r \tag{21e}$$

$$q_{(i,h).\hat{v}_\zeta} = \sum_{j=1}^{N}\zeta_{(i,h,j).\hat{v}_\zeta},\quad \xi_k \triangleq \max\{0, k+1+L-T\} \tag{22}$$

so that $\xi_{T-1} = L$, and $\xi_k = 0$ for all $k < T - L$.

$$\mathcal{X}_{(T,s_i).\varrho_T} = Q. \tag{23}$$

The optimal cost is

$$J(x_0) = x_0^\top\left(\sum_{i=1}^{N}\vartheta_i\,\mathcal{X}_{(0,s_i).\varrho_0}\right)x_0 + \sum_{i=1}^{N}\vartheta_i\,g_{(0,s_i).\varrho_0}, \tag{24}$$

where $\{\vartheta_i\}$ indicates the initial probability distribution of the FSMC's states, and, by convention, $\theta_{-1} = \theta_0$, so that

$$\vartheta_i \triangleq \mathbb{E}\left(1_{\{\theta_0=s_i\}}\right) = \mathbb{E}\left(1_{\{\theta_{-1}=s_i\}}\right), \tag{25}$$

$$g_{(k,s_i).\varrho_k} = \sum_{h=0}^{L-\xi_k}\sum_{j=1}^{N}e_i^\top\varsigma_{(h,j).\hat{v}_\zeta}\left(\sum_{r=1}^{h}\sum_{\nu=0}^{r-1}\mathrm{tr}\left(A^{\nu\top}QA^\nu\Sigma_W\right) + g_{(k+1+h,s_j).\varrho_{k+1+h}} + \sum_{\nu=0}^{h}\mathrm{tr}\left(A^{\nu\top}\mathcal{X}_{(k+1+h,s_j).\varrho_{k+1+h}}A^\nu\Sigma_W\right)\right), \tag{26a}$$

$$g_{(T,s_i).\varrho_T} = 0. \tag{26b}$$

Proof: See Appendix. ■

Remark 4: In (21), (24), and (26a), ϱ_k indicates the combination of the vertices of $Z(k)$ leading to the optimal robust cost. ϱ_{T-1} consists of up to V_s^2 vertices that may produce the terminal cost, ϱ_{T-2} comprises up to $V_s^2V_f$ vertices for each vertex in ϱ_{T-1}, and ϱ_k may have up to V_ζ vertices combined with all those defining subsequent combinations up to ϱ_{k+1+L}. Thus, pruning redundant solutions is essential for limiting the combinatorial growth of involved equations. See, e.g., [25, Sec. 4] and [20, Sec. 3] for pruning methods based on convex combinations and positive semidefiniteness.

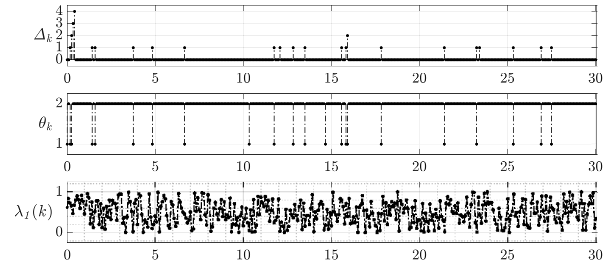

Fig. 1. The PTI FSMC evolution. From top to bottom: Δ_k, θ_k, $\lambda_1(k)$.

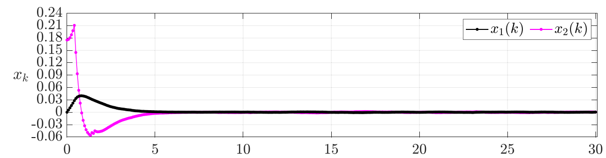

Fig. 2. The system state evolution: $x_k = [x_1(k)\ x_2(k)]^\top$.

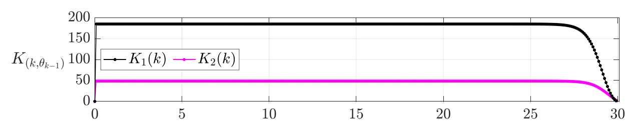

Fig. 3. The selected control gains $K_{(k,\theta_{k-1})} = [K_1(k)\ K_2(k)]$.

IV. Numerical Example

Consider a linearized model of an inverted pendulum on a cart [26]:

$$A = \begin{bmatrix}1.0078 & 0.0301\\ 0.5202 & 1.0078\end{bmatrix},\ B = \begin{bmatrix}-0.0001\\ -0.0053\end{bmatrix}.$$

The cart moves within the 60 cm range perpendicularly to a stationary controller. This movement continuously changes wireless link characteristics. The controller is 5 m from the cart's nominal point and uses ISA-100a communication with a sampling frequency of 12 Hz. The two extreme FSMCs derived as in [18] correspond to the minimal and maximal distances between communicating parties:

$$P_{c.1} = \begin{bmatrix}0.069600 & 0.930400\\ 0.044755 & 0.955245\end{bmatrix},\ P_{c.2} = \begin{bmatrix}0.070018 & 0.929982\\ 0.045070 & 0.954930\end{bmatrix},$$
$$\hat{\delta}_{.1} = \begin{bmatrix}0.082166 & 0.993864\end{bmatrix},\ \hat{\delta}_{.2} = \begin{bmatrix}0.082068 & 0.993828\end{bmatrix}.$$

Let $T = 30$ s. Fig. 1 shows an example of the corresponding PTI FSMC evolution and message dropout count.

Let $\epsilon = 10^{-7}$ so that $L = 5$, and $V_s^2V_f^L = 128$. Consider the following weighting and noise covariance matrices.

$$Q = \begin{bmatrix}0.5 & 0\\ 0 & 4\end{bmatrix},\ R = 0.008,\ \Sigma_w = 10^{-9}\begin{bmatrix}2.5 & 0\\ 0 & 2.5\end{bmatrix}.$$

Fig. 2 shows the evolution of the closed-loop system state under the proposed control law obtained from Theorem 1 for $\phi = 0.5$. After pruning the redundant solutions, as in [25], the controller must choose between only two gains at each time step. E.g., $K_{(1,1)} = [184.899882\ 48.596296]$, and $K_{(1,2)} = [184.897046\ 48.595595]$. Fig. 3 shows the selected control gains.

V. Conclusion

This letter introduced a PTI FSMC model of lossy communication links used in wireless networked control systems and analytically solved the related robust LQR problem under generalized packet dropout compensation. Possible future work includes addressing the remote system state estimation, imperfections in the communication channel state estimation, infinite-horizon issues, and more complex control strategies such as constrained model predictive control.

APPENDIX

Proof of Lemma 2: Note from (2)–(4) that $P_s(k)$ and $P_f(k)$ are always strictly substochastic. In other words, if $P_{l|ij}(k)$ indicates the (i, j) element of $P_l(k)$, with $l \in \{s, f\}$, then $P_{l|ij}(k) \geq 0$, and $\sum_{j=1}^{N} P_{l|ij}(k) < 1$, $\forall k \in \mathbb{Z}^{0+}$, $i, j, N \in \mathbb{Z}^{+}$, and $i, j \leq N$. The matrix product of strictly substochastic matrices is itself strictly substochastic [27, Th. 2.2] and, thus, has a spectral radius of less than one [28, Th. 8.1.22]. Consequently, under Assumption 1, the joint spectral radius of the finite set of matrices $\{P_{l.v_l}\}_{v_l=1}^{V_l}$ is less than one [29, Th. 1], [30], and all matrix products of length n among matrices defined by (5) converge to the zero matrix as $n \to \infty$ [31, Th. I(b)]. Specifically, if $\mathbf{0}$ indicates the zero matrix of size $N \times N$,

$$\lim_{n\to\infty} \prod_{t=1}^{n} P_f\big(\tau_{(m)} + t\big) = \mathbf{0} \ \forall \tau_{(m)} \in \mathbb{Z}^{0+}. \tag{27}$$

Furthermore, all these infinite products converge uniformly at a geometric rate [32, Th. 4.1]. From (17) and (27), for big enough values $\hat{n}$ of a packet error burst $\Delta_{\tau_{(m+1)}}$, the probabilities $\zeta_{(\tau_{(m)},i,\hat{n},j)}$ become negligible, and (18) follows.

Proof of Lemma 3: The result follows from (5), (17), (19), the distributive property of the matrix product, and the commutative property of the product of matrices and real scalars. Formally, $Z(k) =$

$$\sum_{v_1=1}^{V_s} \sum_{v_2=1}^{V_f} \cdots \sum_{v_{L+1}=1}^{V_f} \sum_{v_{L+2}=1}^{V_s} \prod_{d=0}^{L+1} \lambda_{v_{1+d}}(k+d) \left[\frac{e_i^\top P_{s.v_1} \prod_{t=1}^{n} P_{f.v_{t+1}} e_j e_j^\top P_{s.v_{n+2}} \mathbf{1}}{\sum_{v_1=1}^{V_s} \lambda_{v_1}(k) e_i^\top P_{s.v_1} \mathbf{1}} \right]_{i,\gamma(n,j)=1}^{N,(L+1)N}$$

i.e., a convex combination of up to $V_s^2 V_f^L$ matrices.

Proof of Theorem 1: We use the *dynamic programming* approach in Bellman's optimization formulation [33]. $\forall k \in \mathbb{Z}^{0+}$, $k < T$, let $\mathbf{u}_{T-k}^c \triangleq (u_t^c)_{t=k}^{T-1}$ be a control law defined by a sequence of $T-k$ control inputs satisfying (7), and $\mathcal{U}_{T-k}$ be the set of all such control laws. For $l \in \{s, f\}$, let $\mathbf{P}_{T-k} \triangleq (P_{l|i\bullet}(t))_{t=k+1}^{T}$ denote a control message transmission success and failure probability sequence of length $T-k$ and $\mathcal{P}_{T-k}$ indicate the set of all such sequences. Then, we define the cost-to-go as $J(x_k, \theta_{k-1}) = \min_{\mathbf{u}_{T-k}^c \in \mathcal{U}_{T-k}} \max_{\mathbf{P}_{T-k} \in \mathcal{P}_{T-k}} J(x_k, \theta_{k-1}, \mathcal{U}_{T-k}, \mathcal{P}_{T-k})$,

$$J\left(x_k, \theta_{k-1}, \mathcal{U}_{T-k}, \mathcal{P}_{T-k}\right) \triangleq \mathbb{E}\left(\sum_{t=k}^{T-1} \left(x_t^\top Q x_t + u_t^\top R u_t \right) + x_T^\top Q x_T \mid \mathcal{I}_k \right) \tag{28}$$

so that $k = 0$ provides the optimal cost: from (6), (25), (28), and the tower property of the conditional expectation, $J(x_0) = \mathbb{E}(J(x_0, \theta_{-1}) \mid x_0) = \sum_{i=1}^{N} \vartheta_i J(x_0, s_i)$.

We rely on the *backward induction* to obtain the explicit expressions of the cost-to-go and the related optimal robust state-feedback law. We consider the following cost-to-go:

$$J(x_k, \theta_{k-1}) = x_k^\top \mathcal{X}_{(k,\theta_{k-1}).\varrho_k} x_k + g_{(k,\theta_{k-1}).\varrho_k}, \tag{29}$$

where $\mathcal{X}_{(k,\theta_{k-1}).\varrho_k} \succeq 0$ and $g_{(k,\theta_{k-1}).\varrho_k} \geq 0$. The optimal law $\check{\mathbf{u}}_T^c$ from (8) provides the following *terminal cost*. From (6),

$$J(x_T, \theta_{T-1}) = \mathbb{E}\Big(x_T^\top Q x_T \mid \mathcal{I}_T\Big) = x_T^\top Q x_T. \tag{30}$$

Hence, (29) holds in the base case. With $\theta_{T-1} = s_i$, (30) implies (23) and (26b) for any $s_i \in \mathcal{S}$.

For the *induction step*, we assume (29) holds for

$$k + 1 = \min\{\tau_{(m+1)}, T\}, \tag{31}$$

where $\tau_{(m+1)}$ indicates the time instance of the first successful control message transmission following an arbitrary k that may exceed the time horizon T. Let $\tau_{(m)}$ indicate the time instance of the last successful control message transmission preceding $k+1$. From (12) and (31),

$$k = \min\{\tau_{(m)} + \Delta_{\tau_{(m+1)}},\ T - 1\}, \tag{32}$$

where $\Delta_{\tau_{(m+1)}}$ is a discrete stochastic variable. From (28) and the tower property of the conditional expectation,

$$J\big(x_{\tau_{(m)}}, \theta_{\tau_{(m)}-1}\big) = \min_{\mathbf{u}_{T-\tau_{(m)}}^c \in \mathcal{U}_{T-\tau_{(m)}}} \ \max_{\mathbf{P}_{T-\tau_{(m)}} \in \mathcal{P}_{T-\tau_{(m)}}} \mathbb{E}\left(\sum_{t=\tau_{(m)}}^{k} (x_t^\top Q x_t + u_t^\top R u_t) + J(x_{k+1}, \theta_k) \mid \mathcal{I}_{\tau_{(m)}} \right).$$

From w_k being a white Gaussian process with zero mean and covariance matrix Σ_w in (1), the inductive hypothesis, linearity of the expectation, (6), (7), (12)–(14), (32), Assumption 2, and the cyclic property of the trace, we have (33), shown at the top of the page.

Notice that $\tau_{(m)} \leq T - 1$. From (18) and (32), $k - \tau_{(m)}$ is a bounded discrete stochastic variable: $\min(k - \tau_{(m)}) = 0$ and $\max(k - \tau_{(m)}) = \min\{L, T - 1 - \tau_{(m)}\}$. Thus, let $\xi_{\tau_{(m)}} \triangleq \max\{0, \tau_{(m)} + 1 + L - T\}$ so that $\max(k - \tau_{(m)}) = L - \xi_{\tau_{(m)}}$. Then, from (15)–(17) and Remark 1, $\forall\, \mathbf{P}_{T-\tau_{(m)}} \in \mathcal{P}_{T-\tau_{(m)}}$, the matrix differentiation of (33) in $K_{(\tau_{(m)},s_i)}$ leads to (21)–(22), and (26a), with $\tau_{(m)}$, $\zeta_{(\tau_{(m)},i,n,j)}$, $q_{(\tau_{(m)},i,n)}$, and $\varsigma_{(\tau_{(m)},n,j)}$ instead of k, $\zeta_{(i,n,j).\hat{v}_\zeta}$, $q_{(i,n).\hat{v}_\zeta}$, and $\varsigma_{(n,j).\hat{v}_\zeta}$. From Assumption 1 and Lemma 3, all $\mathbf{P}_{T-\tau_{(m)}}$, $\zeta_{(\tau_{(m)},i,n,j)}$, $q_{(\tau_{(m)},i,n)}$, and $\varsigma_{(\tau_{(m)},n,j)}$ are PTI. By the distributive property of the matrix product, commutative property of the product of matrices and real scalars, (5), and (20), Jensen's inequality [34, Th. 4.3] holds for the cost-to-go in $\tau_{(m)}$ taken as a function of only $\mathbf{P}_{T-\tau_{(m)}}$. Consequently, the cost-to-go is a convex function in variable $\mathbf{P}_{T-\tau_{(m)}}$, that belongs to a polytopic set. By [34, Th. 32.2], the maximum in $\mathbf{P}_{T-\tau_{(m)}}$ of the cost-to-go is attained on a vertex of the convex polytope defining $\mathbf{P}_{T-\tau_{(m)}}$. From Lemma 3 and Remark 2, the relevant vertex is among V_ζ vertices that define $Z(\tau_{(m)})$, $\zeta_{(i,n,j).v_\zeta}$, $q_{(i,n).v_\zeta}$ and $\varsigma_{(n,j).v_\zeta}$. So, let $\varrho_{\tau(m)}$ indicate the combination of the vertex $\hat{v}_\zeta$ achieving the cost-to-go at $\tau_{(m)}$ and the vertices providing the cost-to-go in the following steps up to $k+1$. At each transmission time, the controller selects the gain (21a), assuming the actuators receive it successfully, and $\tau_{(m)}$ formalizes this assumption via (12a). Control inputs preceding the current one account for the possibility of its unsuccessful transmission. Rewriting the cost-to-go in k instead of $\tau_{(m)}$ results in (29), with its components defined by (21), (22), and (26a). From (34), shown at the top of the page, $\mathcal{X}_{(k,s_i).\varrho_k} \succeq 0$. As a covariance matrix, $\Sigma_w \succeq 0$, so $g_{(k,s_i).\varrho_k} \geq 0$ since the trace of the product of positive semi-definite matrices is nonnegative.

$$
\begin{aligned}
J\big(x_{\tau_{(m)}}, \theta_{\tau_{(m)}-1} = s_i\big) = \min_{K_{(\tau_{(m)}, s_i)}} \max_{\mathbf{P}_{T-\tau_{(m)}} \in \mathcal{P}_{T-\tau_{(m)}}} \Bigg(x_{\tau_{(m)}}^{\top} \Bigg(\mathbb{E}\Bigg(\sum_{r=1}^{k-\tau_{(m)}} A^{r\top} Q A^{r} + \Big(A^{k-\tau_{(m)}+1}\Big)^{\top} \mathcal{X}_{(k+1,\theta_k).\varrho_{k+1}} A^{k-\tau_{(m)}+1} \mid \mathcal{I}_{\tau_{(m)}} \Bigg) \\
+ Q + K_{(\tau_{(m)}, s_i)}^{\top} \Bigg(R + \mathbb{E}\Bigg(\sum_{r=1}^{k-\tau_{(m)}} \Phi_{(r-1)}^{\top} Q \Phi_{(r-1)} + \phi^{2r} R + \Phi_{(k-\tau_{(m)})}^{\top} \mathcal{X}_{(k+1,\theta_k).\varrho_{k+1}} \Phi_{(k-\tau_{(m)})} \mid \mathcal{I}_{\tau_{(m)}} \Bigg)\Bigg) K_{(\tau_{(m)}, s_i)} \\
+ \mathbb{E}\Bigg(\Big(A^{k-\tau_{(m)}+1}\Big)^{\top} \mathcal{X}_{(k+1,\theta_k).\varrho_{k+1}} \Phi_{(k-\tau_{(m)})} K_{(\tau_{(m)}, s_i)} + K_{(\tau_{(m)}, s_i)}^{\top} \Phi_{(k-\tau_{(m)})}^{\top} \mathcal{X}_{(k+1,\theta_k).\varrho_{k+1}} A^{k-\tau_{(m)}+1} + \sum_{r=1}^{k-\tau_{(m)}} K_{(\tau_{(m)}, s_i)}^{\top} \Phi_{(r-1)}^{\top} Q A^{r} \\
+ A^{r\top} Q \Phi_{(r-1)} K_{(\tau_{(m)}, s_i)} \mid \mathcal{I}_{\tau_{(m)}} \Bigg)\Bigg) x_{\tau_{(m)}} + \mathbb{E}\Bigg(\sum_{\nu=0}^{k-\tau_{(m)}} \operatorname{tr}\Big(A^{\nu\top} \mathcal{X}_{(k+1,\theta_k).\varrho_{k+1}} A^{\nu} \Sigma_W \Big) + \sum_{r=1}^{k-\tau_{(m)}} \sum_{\nu=0}^{r-1} \operatorname{tr}(A^{\nu\top} Q A^{\nu} \Sigma_W) + g_{(k+1,\theta_k).\varrho_{k+1}} \mid \mathcal{I}_{\tau_{(m)}} \Bigg)\Bigg)
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
\mathcal{X}_{(k,s_i).\varrho_k} = Q + \sum_{h=0}^{L-\xi_k} q_{(i,h).\hat{v}_\zeta} \sum_{r=1}^{h} \big(A^{r} + \Phi_{(r-1)} K_{(k,s_i)}\big)^{\top} Q \big(A^{r} + \Phi_{(r-1)} K_{(k,s_i)}\big) + K_{(k,s_i)}^{\top} \Bigg(R + \sum_{h=0}^{L-\xi_k} q_{(i,h).\hat{v}_\zeta} \sum_{r=1}^{h} \phi^{2r} R \Bigg) K_{(k,s_i)} \\
+ \sum_{h=0}^{L-\xi_k} \sum_{j=1}^{N} \zeta_{(i,h,j).\hat{v}_\zeta} \Big(A^{h+1} + \Phi_{(h)} K_{(k,s_i)} \Big)^{\top} \mathcal{X}_{(k+h+1,s_j).\varrho_{k+h+1}} \Big(A^{h+1} + \Phi_{(h)} K_{(k,s_i)} \Big)
\end{aligned} \tag{34}
$$

REFERENCES

[1] P. Park, S. Ergen, C. Fischione, C. Lu, and K. Johansson, "Wireless network design for control systems: A survey," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 2, pp. 978–1013, 2nd Quart., 2018.

[2] M. Eisen, M. Rashid, K. Gatsis, D. Cavalcanti, N. Himayat, and A. Ribeiro, "Control aware radio resource allocation in low latency wireless control systems," *IEEE Internet Things J.*, vol. 6, no. 5, pp. 7878–7890, Oct. 2019.

[3] M. Pezzutto, F. Tramarin, S. Dey, and L. Schenato, "Adaptive transmission rate for LQG control over Wi-Fi: A cross-layer approach," *Automatica*, vol. 119, Sep. 2020, Art. no. 109092.

[4] W. Liu, G. Nair, Y. Li, D. Nesic, B. Vucetic, and H. Poor, "On the latency, rate, and reliability tradeoff in wireless networked control systems for IIoT," *IEEE Internet Things J.*, vol. 8, no. 2, pp. 723–733, Jan. 2021.

[5] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. Sastry, "Foundations of control and estimation over lossy networks," *Proc. IEEE*, vol. 95, no. 1, pp. 163–187, Jan. 2007.

[6] V. Gupta, A. Dana, J. Hespanha, R. Murray, and B. Hassibi, "Data transmission over networks for estimation and control," *IEEE Trans. Autom. Control*, vol. 54, no. 8, pp. 1807–1819, Aug. 2009.

[7] W. Heemels, A. Teel, N. van de Wouw, and D. Nesic, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1781–1796, Aug. 2010.

[8] A. Gonçalves, A. Fioravanti, and J. Geromel, "Markov jump linear systems and filtering through network transmitted measurements," *Signal Process.*, vol. 90, no. 10, pp. 2842–2850, 2010.

[9] B. Ding, "Stabilization of linear systems over networks with bounded packet loss and its use in model predictive control," *Automatica*, vol. 47, no. 11, pp. 2526–2533, 2011.

[10] M. Pajic, S. Sundaram, G. Pappas, and R. Mangharam, "The wireless control network: A new approach for control over networks," *IEEE Trans. Autom. Control*, vol. 56, no. 10, pp. 2305–2318, Oct. 2011.

[11] P. Minero, L. Coviello, and M. Franceschetti, "Stabilization over Markov feedback channels: The general case," *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 349–362, Feb. 2013.

[12] D. Quevedo, J. Ostergaard, and A. Ahlén, "Power control and coding formulation for state estimation with wireless sensors," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 2, pp. 413–427, Mar. 2014.

[13] J.-T. Yu and L.-C. Fu, "An optimal compensation framework for linear quadratic Gaussian control over lossy networks," *IEEE Trans. Autom. Control*, vol. 60, no. 10, pp. 2692–2697, Oct. 2015.

[14] Y. Zacchia Lun and A. D'Innocenzo, "Stabilizability of Markov jump linear systems modeling wireless networked control scenarios," in *Proc. IEEE Conf. Decis. Control (CDC)*, 2019, pp. 5766–5772.

[15] A. Impicciatore, A. Tsiamis, Y. Zacchia Lun, A. D'Innocenzo, and G. Pappas, "Secure state estimation over Markov wireless communication channels," in *Proc. IEEE Conf. Decision Control (CDC)*, 2022, pp. 2935–2940.

[16] A. Impicciatore, Y. Zacchia Lun, P. Pepe, and A. D'Innocenzo, "Optimal output-feedback control over Markov wireless communication channels," *IEEE Trans. Autom. Control*, vol. 69, no. 3, pp. 1643–1658, Mar. 2024.

[17] P. Sadeghi, R. Kennedy, P. Rapajic, and R. Shams, "Finite-state Markov modeling of fading channels—A survey of principles and applications," *IEEE Signal Process. Mag.*, vol. 25, no. 5, pp. 57–80, Sep. 2008.

[18] Y. Zacchia Lun, C. Rinaldi, A. D'Innocenzo, and F. Santucci, "Co-designing wireless networked control systems on IEEE 802.15.4-based links under Wi-Fi interference," *IEEE Access*, vol. 12, pp. 71157–71183, 2024.

[19] Y. Zacchia Lun, A. D'Innocenzo, A. Abate, and M. D. Di Benedetto, "Optimal robust control and a separation principle for polytopic time-inhomogeneous Markov jump linear systems," in *Proc. IEEE Conf. Decision Control (CDC)*, 2017, pp. 6525–6530.

[20] Y. Zacchia Lun, A. Abate, and A. D'Innocenzo, "Linear quadratic regulation of polytopic time-inhomogeneous Markov jump linear systems," in *Proc. Eur. Control Conf. (ECC)*, 2019, pp. 4094–4099.

[21] Y. Zacchia Lun, A. D'Innocenzo, and M. D. Di Benedetto, "Robust stability of polytopic time-inhomogeneous Markov jump linear systems," *Automatica*, vol. 105, pp. 286–297, Jul 2019.

[22] M. Moayedi, Y. K. Foo, and Y. C. Soh, "Networked LQG control over unreliable channels," *Int. J. Robust Nonlinear Control*, vol. 23, no. 2, pp. 167–189, 2013.

[23] Z. Lu, S. Zhong, and L. Qu, "A switching approach to packet loss compensation strategy," *IEEE Access*, vol. 7, pp. 5609–5615, 2018.

[24] D. J. Hartfiel, *Markov Set-Chains* (Lecture Notes in Mathematics), vol. 1695. Berlin, Germany: Springer-Verlag, 1998.

[25] Y. Zacchia Lun, A. D'Innocenzo, and M. Di Benedetto, "Robust LQR for time-inhomogeneous Markov jump switched linear systems," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 2199–2204, 2017.

[26] X.-M. Zhang, Q.-L. Han, and X. Ge, "A novel approach to H_∞ performance analysis of discrete-time networked systems subject to network-induced delays and malicious packet dropouts," *Automatica*, vol. 136, Feb. 2022, Art. no. 110010.

[27] L. Shi, W. Zheng, J. Shao, and Y. Cheng, "Sub-/super-stochastic matrix with applications to bipartite tracking control over signed networks," *SIAM J. Control Optim.*, vol. 59, no. 6, pp. 4563–4589, 2021.

[28] R. Horn and C. Johnson, *Matrix Analysis*, 2nd ed. Cambridge, MA, USA: Cambridge Univ. Press, 2012.

[29] M.-H. Shih, J.-W. Wu, and C.-T. Pang, "Asymptotic stability and generalized gelfand spectral radius formula," *Linear Algebra Appl.*, vol. 252, nos. 1–3, pp. 61–70, 1997.

[30] N. E. Barabanov, "On the Lyapunov exponents of discrete inclusions," *Autom. Remote Control*, vol. 49, no. 2, pp. 40–46, 1988.

[31] M. A. Berger and Y. Wang, "Bounded semigroups of matrices," *Linear Algebra Appl.*, vol. 166, pp. 21–27, Mar. 1992.

[32] I. Daubechies and J. C. Lagarias, "Sets of matrices all infinite products of which converge," *Linear Algebra Appl.*, vol. 161, pp. 227–263, Jan. 1992.

[33] D. P. Bertsekas, *Dynamic Programming and Optimal Control*. London, U.K.: Athena Sci., 1995.

[34] R. T. Rockafellar, *Convex Analysis*. Princeton, NJ, USA: Princeton Univ. Press, 1997.

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